Side-Channel Analysis of Multiplications in $GF(2^{128})$

Application to AES-GCM

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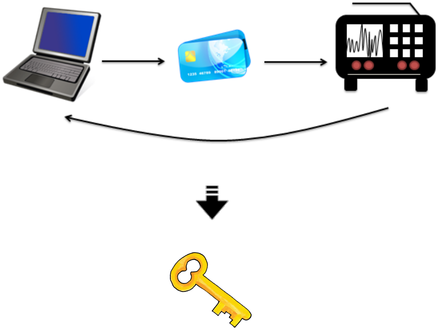
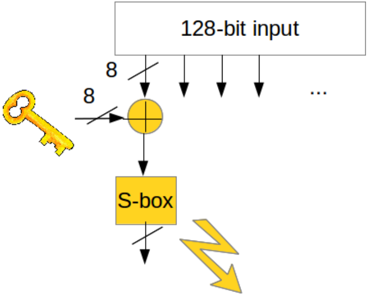
Side-Channel Attacks

- physical leakage
  - timing
  - power consumption
  - temperature
  - ...
- statistical treatment
- key recovery



Key-Dependent Leakage

AES Block Cipher



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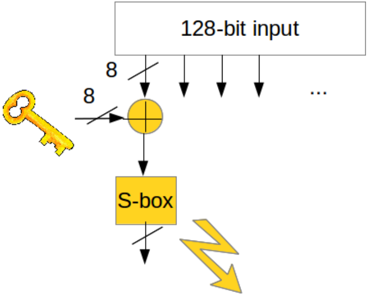
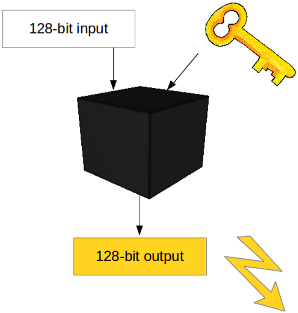
AES Block Cipher



GCM : Multiplication in $GF(2^{128})$



Outline

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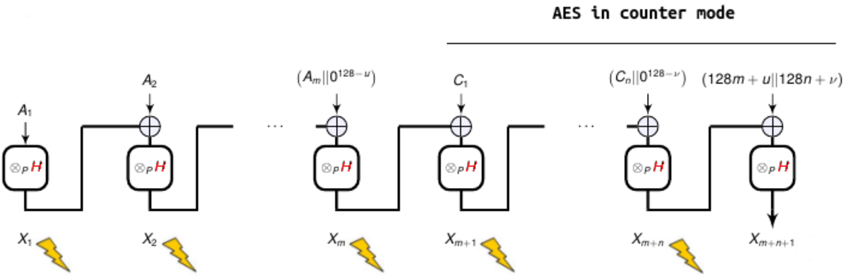
AES-GCM



hashed key H: $H = \mathrm{AES}_K(0^{128})$ with K the encryption key

authenticated data A_i: 128-bit blocks of data to authenticate

ciphertexts C_i: 128-bit encrypted blocks

Galois Field Multiplication $\otimes_P$

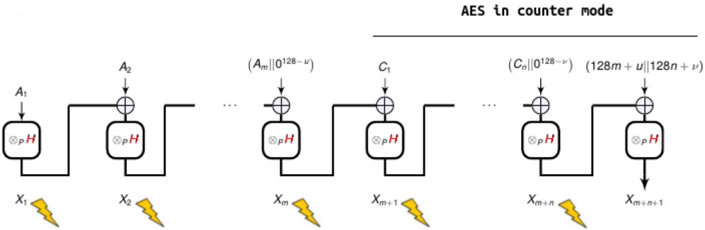
$$\mathrm{GF}(2^{128}) = \mathrm{GF}(2)[Y]/P(Y), \quad P(Y) = Y^{128} + Y^7 + Y^2 + Y + 1$$

$$M_P \cdot H =$$

$$\begin{pmatrix} m_0 & m_{127} & \cdots & m_1 \oplus m_{127} \oplus m_{126} \\ m_1 & m_0 \oplus m_{127} & \cdots & m_2 \oplus m_{123} \oplus m_1 \oplus m_{127} \oplus m_{122} \\ \vdots & \vdots & \ddots & \vdots \\ m_{127} & m_{126} & \cdots & m_0 \oplus m_{127} \oplus m_{126} \oplus m_{121} \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ \vdots \\ h_{127} \end{pmatrix}$$

$$\underbrace{\phantom{m_{127}}}_{M \otimes_P \alpha^0} \; \underbrace{\phantom{m_0 \oplus m_{127}}}_{M \otimes_P \alpha^1} \quad \cdots \quad \underbrace{\phantom{m_0 \oplus m_{127} \oplus m_{126} \oplus m_{121}}}_{M \otimes_P \alpha^{127}}$$

Leakage Models



Hamming Weight

$$L_i^{(\mathrm{HW})} = \mathrm{HW}(X_i) + \varepsilon_\sigma, \quad \varepsilon_\sigma \sim \mathcal{N}(0, \sigma)$$

Hamming Distance

$$L_i^{(\mathrm{HD})} = \mathrm{HD}(X_i, X_{i-1}) + \varepsilon_\sigma = \mathrm{HW}(X_i \oplus X_{i-1}) + \varepsilon_\sigma$$

Outline

Context

Attack
- Main Idea
- Known Inputs
- Chosen Inputs

Conclusion

Outline

Main Idea of The Attack

Current Issue: each bit of the 128-bit multiplication's result depends on all the key bits

- ✘ no divide-and-conquer strategy

Main observation: the LSB of the Hamming Weight (same for HD) of a variable is a linear function of its bits:

$$\mathrm{lsb}_0\left(\mathrm{HW}(V)\right) = \bigoplus_{0 \leqslant i \leqslant 127} v_i$$

LSB of the first multiplication output's Hamming weight:

$$b_0 \stackrel{\text{def}}{=} \mathrm{lsb}_0\left(\mathrm{HW}(M \otimes_P H)\right) = \bigoplus_{0 \leqslant i \leqslant 127} (M \otimes_P H)_i$$

$$= \bigoplus_{0 \leqslant j \leqslant 127} \left(\bigoplus_{0 \leqslant i \leqslant 127} (M_P)_{i,j} \right) h_j$$

Linear system to solve:

$$\mathcal{S} = \begin{cases} \bigoplus_{0 \leqslant j \leqslant 127} \left(\bigoplus_{0 \leqslant i \leqslant 127} (M_P^{(0)})_{i,j} \right) h_j = {b_0}^{(0)} \\ \bigoplus_{0 \leqslant j \leqslant 127} \left(\bigoplus_{0 \leqslant i \leqslant 127} (M_P^{(1)})_{i,j} \right) h_j = {b_0}^{(1)} \\ \qquad \cdots \\ \bigoplus_{0 \leqslant j \leqslant 127} \left(\bigoplus_{0 \leqslant i \leqslant 127} (M_P^{(t-1)})_{i,j} \right) h_j = {b_0}^{(t-1)} \end{cases}$$

New Issue

New Issue: leakage comes with noise

$$\widetilde{b_0} \stackrel{\text{def}}{=} \text{lsb}_0\left(\lceil \text{HW}(M \otimes_P H) + \varepsilon_\sigma \rfloor\right) = \text{lsb}_0\left(\text{HW}(M \otimes_P H)\right) \oplus b_{\mathcal{N}}$$

Probability of error on $b_{\mathcal{N}}$: $p_\sigma = 1 - \sum_{i=-\infty}^{\infty} \int_{2i-0.5}^{2i+0.5} \phi_\sigma(t)dt$

$$\begin{array}{lcl} \sigma = 0.5 & \rightarrow & p_\sigma = 0.31 \\ \sigma = 1 & \rightarrow & p_\sigma = 1/2 - 4.6\ 10^{-3} \\ \sigma = 2 & \rightarrow & p_\sigma = 1/2 - 1.7\ 10^{-9} \\ \sigma \geqslant 3 & \rightarrow & p_\sigma = 1/2 - \varepsilon \end{array}$$

Outline

Naive Attack

$$\widetilde{\mathcal{S}} = \begin{cases} \bigoplus_{0 \leqslant j \leqslant 127} \left(\bigoplus_{0 \leqslant i \leqslant 127} (M_P^{(0)})_{i,j} \right) h_j = \widetilde{b_0}^{(0)} \\ \bigoplus_{0 \leqslant j \leqslant 127} \left(\bigoplus_{0 \leqslant i \leqslant 127} (M_P^{(1)})_{i,j} \right) h_j = \widetilde{b_0}^{(1)} \\ \qquad \qquad \cdots \\ \bigoplus_{0 \leqslant j \leqslant 127} \left(\bigoplus_{0 \leqslant i \leqslant 127} (M_P^{(t-1)})_{i,j} \right) h_j = \widetilde{b_0}^{(t-1)} \end{cases}$$

Naive attack:

i) extract 128 equations linearly independent

ii) remove the errors on bits $\widetilde{b_0}^{(\ell)}$ by enumeration

Improvements

- Reducing the Noise Impact
- Saving Executions
- Solving the System with Dedicated Algorithms

An Optimal Decision Rule

First Idea: use the LLR (Log Likelihood Ratio) to approximate better the bit value b_0

$$\widehat{b_0} \stackrel{\text{def}}{=} \begin{cases} 0 & \text{if } \mathrm{LLR}(\ell) \geqslant 0, \\ 1 & \text{otherwise.} \end{cases}$$

with

$$\mathrm{LLR}(\ell) = \log(\ \mathbb{P}[b_0 = 0 \mid \ell]\ ) - \log(\ \mathbb{P}[b_0 = 1 \mid \ell]\ )$$

Second Idea: when more than 128 traces are available, choose 128 linearly independent samples from the highest LLR values

Selecting Traces

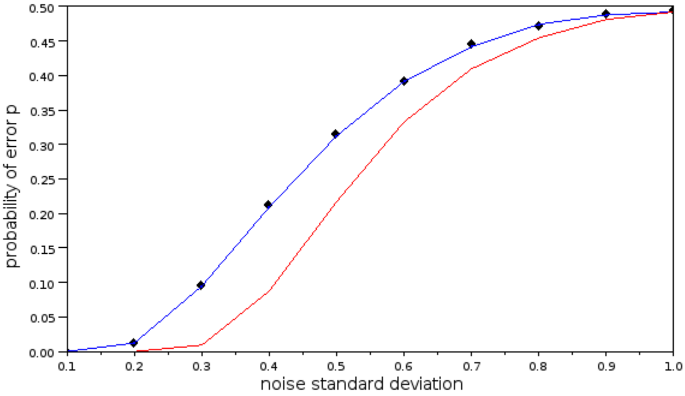
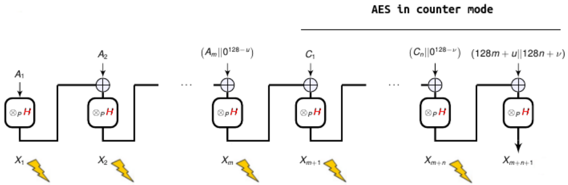


Figure: Error probability with rounding (black), LLR (blue) and best LLRs (red)

Saving Executions



Second Multiplication:

$$
\begin{aligned}
X_2 &= (M_1 \otimes_P H \oplus M_2) \otimes_P H \\
&= M_1 \otimes_P H^2 \oplus M_2 \otimes_P H
\end{aligned}
$$

Since squaring is linear over GF(2), there exists S such that

$$X_2 = (M_1 \otimes_P S \oplus M_2) \otimes_P H$$

- two multiplications with a single execution

Solving the System with Dedicated Algorithms

Noisy codeword: LSBs extracted from leaking multiplications that encode the authentication key H

Issue: decoding the noisy codeword

- Learning Parities with Noise (LPN) Algorithms
- Linear Decoding

Method \ σ	0.1 C_s/C_t	0.2 C_s/C_t	0.3 C_s/C_t	0.4 C_s/C_t	0.5 C_s/C_t
LLR + naive	$2^8/2^{21}$	$2^8/2^{21}$	$2^8/2^{22}$	$2^8/2^{65}$	$2^8/2^{107}$
LPN (LF Algo)	$2^{11}/2^{14}$	$2^{20}/2^{22}$	$2^{26}/2^{28}$	$2^{32}/2^{34}$	$2^{48}/2^{50}$
Linear decoding	$2^6/2^6$	$2^6/2^7$	$2^7/2^{11}$	$2^8/2^{25}$	$2^9/2^{62}$

Outline

Improvements

- Averaging the traces
- Structuring the messages to make the system easier to solve
- Choosing messages to exploit more than two multiplications in a single execution

Averaging Traces

Repeating the same computation λ times: $\sigma \mapsto \sigma/\sqrt{\lambda}$

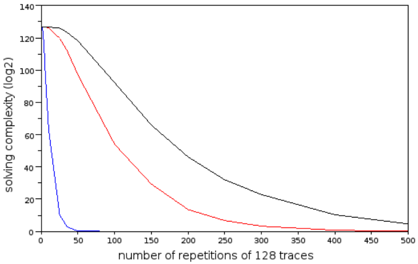


Figure: Solving complexities with repetitions for $\sigma = 1$ (blue), $\sigma = 3$ (red) and $\sigma = 4$ (black)

Experimental Results: tests on the Virtex-5 FPGA of a SASEBO board with an EM probe for the acquisition

- confirm the simulations

Structuring the Messages

Current Issue: the linear code corresponding to our attack is random and have a high dimension (128)

Better Code: concatenation of smaller random linear codes

- with the enumeration algorithm from [1], an attacker can enumerate keys from ordered lists of key chunks
- each block corresponds to a smaller linear code that may be fully decoded by a Fast Walsh Transform.

$$\begin{pmatrix} \boxed{\mathcal{S}_0} & & \\ & \boxed{\mathcal{S}_1} & \\ & & \ddots \end{pmatrix} \cdot \begin{pmatrix} H \end{pmatrix} = \begin{pmatrix} \widehat{b}_0 \\ \vdots \\ \widehat{b}_t \end{pmatrix}$$

[1] Veyrat-Charvillon, Gérard, Renauld, and Standaert. *An optimal key enumeration algorithm and its application to side-channel attacks.* In SAC 2012,LNCS, pages 390–406.

Structuring the Messages

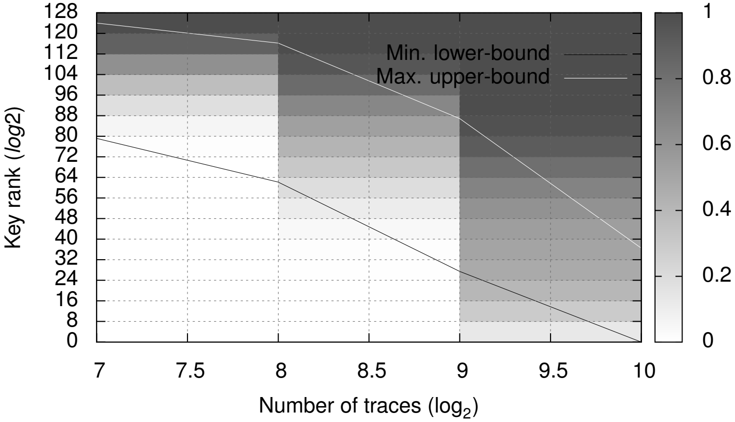


Figure: Security graph[2] for $\sigma = 0.5$

[2] Veyrat-Charvillon, Gérard, and Standaert. *Security evaluations beyond computing power.* In EUROCRYPT 2013,LNCS, pages 126–141.

Saving Executions

Saving Executions: exploit the linearity of the squaring operation (as suggested by Ferguson)

$$
\begin{aligned}
X_1 &= M_1 \otimes_P H, \\
X_2 &= M_1 \otimes_P H^2 \oplus M_2 \otimes_P H, \\
X_3 &= M_1 \otimes_P H^3 \oplus M_2 \otimes_P H^2 \oplus M_3 \otimes_P H, \\
X_4 &= M_1 \otimes_P H^4 \oplus M_2 \otimes_P H^3 \oplus M_3 \otimes_P H^2 \oplus M_4 \otimes_P H.
\end{aligned}
$$

Saving Executions

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X_3 &= M_1 \otimes_P H^3 \oplus M_2 \otimes_P H^2 \oplus M_3 \otimes_P H, \\
X_4 &= M_1 \otimes_P H^4 \oplus M_2 \otimes_P H^3 \oplus M_3 \otimes_P H^2 \oplus M_4 \otimes_P H.
\end{aligned}$$

$$M_2 = 0$$

Saving Executions

Saving Executions: exploit the linearity of the squaring operation (as suggested by Ferguson)

- $X_1 = M_1 \otimes_P H,$
- $X_2 = M_1 \otimes_P H^2,$

  $X_3 = M_1 \otimes_P H^3 \oplus M_3 \otimes_P H,$
- $X_4 = M_1 \otimes_P H^4 \oplus M_3 \otimes_P H^2 \oplus M_4 \otimes_P H.$

$$M_2 = 0$$

Outline

Conclusion

- Summary
  - attack the AES-GCM authentication without looking inside the multiplication
  - exploitation of the LSB
  - different improvements
- Further Work
  - application of similar attacks to other primitives
  - exploitation of more leakage bits with different techniques

Thank you

Thank you for your attention.

Application on the other bits

$$b_i = \bigoplus_{0 \leqslant j_1 < \cdots < j_{2^i} \leqslant 127} \left(\prod_{1 \leqslant \ell \leqslant 2^i} \bigoplus_{0 \leqslant k \leqslant 127} (M \otimes_P \alpha^k)_{j_\ell} \, h_k \right), \quad \forall\, 0 \leqslant i \leqslant 7$$

σ	Bernoulli parameter p							
	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7
0.5	$3.1\,10^{-1}$	$1.6\,10^{-1}$	$8.0\,10^{-2}$	$4.0\,10^{-2}$	$2.3\,10^{-2}$	$2.2\,10^{-2}$	$2.2\,10^{-2}$	ε
1	$\frac{1}{2} - 4.6\,10^{-3}$	$3.7\,10^{-1}$	$1.9\,10^{-1}$	$9.5\,10^{-2}$	$5.5\,10^{-2}$	$5.3\,10^{-2}$	$5.3\,10^{-2}$	ε
2	$\frac{1}{2} - 1.5\,10^{-4}$	$\frac{1}{2} - 3.2\,10^{-3}$	$3.8\,10^{-1}$	$2.0\,10^{-1}$	$1.1\,10^{-1}$	$1.1\,10^{-1}$	$1.1\,10^{-1}$	ε
3	$\frac{1}{2} - \varepsilon$	$\frac{1}{2} - 6.8\,10^{-8}$	$4.7\,10^{-1}$	$3.0\,10^{-1}$	$1.6\,10^{-1}$	$1.5\,10^{-1}$	$1.5\,10^{-1}$	ε
4	$\frac{1}{2} - \varepsilon$	$\frac{1}{2} - 1.2\,10^{-9}$	$\frac{1}{2} - 3.0\,10^{-3}$	$3.8\,10^{-1}$	$2.1\,10^{-1}$	$1.9\,10^{-1}$	$1.9\,10^{-1}$	ε
5	$\frac{1}{2} - \varepsilon$	$\frac{1}{2} - \varepsilon$	$\frac{1}{2} - 1.9\,10^{-4}$	$4.4\,10^{-1}$	$2.6\,10^{-1}$	$2.3\,10^{-1}$	$2.3\,10^{-1}$	ε

Re-keying from Medwed et al.[3]

$$k^\star = r \cdot k \quad \in \quad \mathrm{GF}(2^8)[Y]/P(Y) = Y^{16} + 1$$

Matrix/vector product $K^\star = R_P \otimes_P K$ with

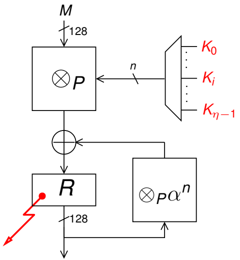
$$R_p = \begin{pmatrix} r_0 & r_{15} & \cdots & r_1 \\ r_1 & r_0 & \cdots & r_2 \\ \vdots & \vdots & \ddots & \vdots \\ r_{15} & r_{14} & \cdots & r_0 \end{pmatrix}$$

Equation of the LSB:

$$\mathrm{lsb}_0 \left(\mathrm{HW} \left[\left(\bigoplus_{0 \leqslant i \leqslant m-1} r_i \right) \cdot \left(\bigoplus_{0 \leqslant j \leqslant m-1} k_j \right) \right] \right) = b_0$$

[3] M. Medwed, C. Petit, F. Regazzoni, M. Renauld, F.-X. Standaert, Fresh Re-Keying II: Securing Multiple Parties against Side-Channel and Fault Attacks, CARDIS 2011

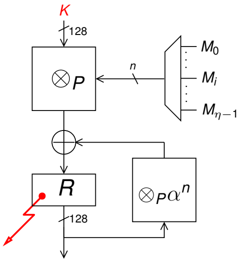
Specific Implementations



if the key is split

- divide-and-conquer strategy

Specific Implementations



if the key is split

- divide-and-conquer strategy

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if the message is split

- sparse messages
- easier than the generic (known inputs) scenario